Optimality of empirical $Z-R$ relations

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Summary

This paper attempts to justify mathematically the two empirical approaches to the problem of deriving Z–R relations from (Z, R) measurements, namely the power-law regression and the "probability matching method" (PMM). The basic mathematical assumptions that apply in each case are explicitly identified. In the first case, the appropriate assumption is that the scatter in the (Z, R) measurements reflects exactly the randomness in the connection between Z and R due to a lack of sufficient a priori information about either of them. In the second case, the assumption is that the measurements have been classified into categories a priori, in a way that allows one to expect a nearly one-to-one correspondence between Z and R in each catergory, the scatter in the measurements being due to residual noise. The paper then shows how the assmuptions naturally lead, in the first case, to a "conditional-mean" Z–R relation of which the power laws are regression based approximations, and, in the second case, to a "probability-matched" relation.

1 Introduction

There are two essentially different approaches to establishing radar-reflectivity ↔ rain-rate relations from experimental data. The first (and original) technique to estimate rainfall using radar measurements is based on the physical relation between the rain parameters (rain rate R, drop size density function) and the radar reflectivity coefficient Z (see e.g. Ryde 1946, Marshall and Palmer 1948). Originally, a simple power law $Z = aR^b$ was used. Subsequent regression analyses of measured data consisting of simultaneous observations of rain intensities and radar reflectivities have produced a plethora of power-law Z–R relations, showing large variations in the value of the coefficient a and the exponent b (see e.g. Battan 1973). Most of these relations were calculated from disdrometer-measured drop size histograms, typically using notoriously biased sample moments to estimate the corresponding values of Z and R. Several authors have proposed subjective classification criteria to reduce these ambiguities (see e.g. Austin 1987). Typical subjective classifying categories include drizzle, thunderstorm rain, wide-spread rain, convective cell rain, etc.

An altogether different second approach to relate Z to R and reduce these ambiguities was originally proposed by Calheiros and Zawadski (1987), and later developed and extended by Atlas and Rosenfeld (see e.g. Atlas et al 1990, Rosenfeld et al. 1994 and 1995b). In its present form, the resulting "probability matching method" (PMM) seeks to classify the type of rain at hand using objective criteria before attempting to derive the appropriate Z–R relation. Rather than expressing the Z–R relation in terms of a given set of rain parameters, the method first classifies the rain regime according to quantitatively robust criteria, then (1(’rives the appropriate Z–R relation directly from *marginal* Z and R data, collected from sample events governed by that particular regime. The resulting relations are therefore no longer given by analytic closed-form expressions, but they are just as efficiently computable as the parametric closed form relations described above.

This paper attempts to justify mathematically the two empirical approaches to the problem of deriving Z–R relations: tll(’])()\\’(’I’-ld\\’ regression and the PMM. In each (’iIS(’. we identify explicitly the basic mathematical assumptions that ii])])])’. and we Silo\ \" in what way they lead to a power-law relation, in the first (’iIS(’, and a "probability-matched" relation in the second. It tiII’IIS out that the starting mathematical hypotheses appropriate to each case are

1) one is given simultaneous (Z, R) measurements from one or more rain events with little or no a priori climatological or physical information.

2) One is given Z- and R- measurements from one or more rain (’\"(’iIIs sharing precise objective climatological, physical and geometric characteristics,

from which one wishes to derive the Z–R relation which best predicts R given Z, for the particular class of rain events at hand.

The main difference is in the amount of effort one decides to expend a priori in classifying the rain events according to the relevant climatological/physical/geometric considerations. In each of the two cases, the guiding principle is to make sure that the Z–R relation derived is optimal according to the mathematical assumptions made. This ensures that, given a particular rain event, the estimates obtained using either one of these two approaches can be rigorously justified without resorting to extraneous assumptions.

The first set of assumptions leads to a conditional-mean Z–R relation, the optimal relation in this case. It is discussed in section 2, where it is then shown how the power laws $Z = aR^b$ are more or less reasonable approximations of the optimal conditional-mean Z–R relation. In section 3, we show how the second set of assumptions naturally leads to the PMM. In this case, the PMM is itself the optimal relation.

We take this opportunity to point out that other approaches, such as the "parametric" (for lack of a better description) method described in Haddad et al (1996a) and applied to estimate rain profiles in Haddad et al (1996b), can be used. We shall confine this discussion to the two empirical methods described above.

2 Z–R using no a priori classification

Assume that a large number of careful simultaneous (Z, R) measurements were collected using disdrometers or radar-and-rain-gauges during one or more typical rain events, and that a sampled joint probability density function $\mathcal{P}_{(Z,R)}$ has been compiled. Without accounting for horizontal and vertical inhomogeneities in the drop distribution, which can cause non-uniformities in the Z- and R- measurements, or for the possible changes in the stratiform/convective nature of the rain event as it evolves in time, Z and R will still be correlated, but there will be no one-to-one correspondence between measured reflectivity levels and single-point rain-rate figures. In this case therefore, it is not reasonable to look for a deterministic Z–R relationship. Rather, the best one can do under these assumptions is to look for that function $R = f(Z)$ which makes, on average, the smallest error as calculated from the observed (Z, R) measurements. It is a basic result of probability theory that $f(z)$ is the conditional mean $\mathcal{E}\{R \mid Z = z\}$ of R given that $Z = z$,

$$f(z) = \mathcal{E}\{R \mid Z = z\} = \frac{\int r\,\mathcal{P}_{(Z,R)}(z,r)\,dr}{\int \mathcal{P}_{(Z,R)}(z,r)\,dr} \qquad (1)$$

Yet, in this case, traditionally, power-law relations of the form Z : aR^b have 1)('('11 11s('(1, How are 1 hey related to the statistically optimal approach '? Practically, the optimal formula above would lead one to infer directly the appropriate relation using the sample conditional mean. However if the conditional III('IIII of R given that Z: does turn out to be a])(JW('I' law function of z (as in the case where (, Y,/t') are jointly lognormal), then one should obtain t 11(' same values for a and b, whether one calculates the power-law parameters a and b directly from the sample conditional mean or by performing a regression on the data. Three problems arise: first and most obvious, if the conditional mean of R given Z is not exactly a power-law function of Z, the power-law regression would produce a correspondingly inaccurate fit. Figure 1 a reproduces an example due to Short et al (1993), showing one instance in which no power law can adequately describe the governing Z R relation. Figure I b silo\\'s the optimal conditional-mean relation for this data, obtained using (1), along with the power-law regression curves corresponding to the two more or less distinct rain regimes governing different segments of the data. Note that in this case, the relative r.m. s. error due to the discretization of the measurements (not including uncertainties due to the limited hardware precision, the instrument design, or the calibration) varies from I o % for R and 12% for Z, in the heavy-rain samples, to I S'X for R and 28% for Z, in the light-rain samples.

A second drawback of using unclassified data to infer a Z-R relation, whether one ('1100 s('S" 10 use a power-law approximation or the sample conditional mean, is that one then 11(eds a number of simultaneous and colocated (Z, R) measurements that is large enough 1 0 build a suitably accurate joint density function $\mathcal{P}_{(Z,R)}$. In practice, it is quite difficult to make measurements of the pair (Z_e, R) that can be considered simultaneous, Z_e denoting the radar-estimated Z over the rain gauge and R the rain-gauge-measured rain intensity. Moreover, collecting a sufficiently large number of samples can affect the homogeneity of t he resulting (Z, R) population, as the example of Fig. 1a illustrates.

Finally, while the conditional-mean approach does produce a Z R relation which, among all possible formulas one can use to relate Z and R, makes the smallest r.m.s. error when the data is unclassified, this error is *not necessarily small*. *In fact*, the ('1'1'01" ('illl be estimated directly from the simultaneous (Z, R) measurements: it is given by the square root of the (conditional) sample variance, i.e. by the scatter in the joint (Z, R) measurements. When this scatter is large, the usefulness of the resulting Z R relation is uncertain. Figure 1 c shows a plot of the relative r.m.s. uncertainty (i .('. of the square root of the conditional variance, divided by the conditional mean), as a function of the radar reflectivity, for the data 01" Fig. 1a. In this case, as Z increases, the relative r.m. s. uncertainty in the optimally-estimated R gradually decreases to about 20% (the subsequent decrease when Z is greater t han 45 dBZ is due mainly to the extremely small sample size). At 31 dBZ, one standard deviation still amounts to an uncomfortably large 50%.

3 Z–R using comprehensive a priori classification

In order to avoid having a Z–R relation based on measurements with a relatively large variance, and, therefore, to avoid making a correspondingly large r.m.s. error in the estimation of R, one might try to carefully classify one's data a priori. In this section, we shall therefore consider a radically different starting hypothesis: assume that a careful climatological classification allows one to characterize the rai events as belonging to well-defined categories such that within each category R and Z can be determined from one another exactly (with probability , i.e. such t at for each ca egory there exists a monotone function f which satisfies R $f(Z)$. This function s med to be derivable from physical (rather than probabilistic) considerations appropriate to each of the rain categories within our hypothetical classification. Thus, unlike the function of the same name in the previous case, this function f has so far nothing to do with probabilities. Suppose now that one wants to determine f not from physical considerations, but rather from measurements of a carefully compiled and classified catalogue of Z and R measurements. How can one use probabilities find f from such a collec ion of samples ?

It turns out that, in this case, simultaneous measurements are not needed. Indeed, suppose that we have identified the rain regime of interest, and that we have enough Z- and R- measurements from events that fall within this regime to construct their respective probability density functions $\mathcal{P}_Z$ and $\mathcal{P}_R$ (note that this requires much fewer samples than in the previous approach). Indeed, since $f(Z) = R$,

$$\int_0^z \mathcal{P}_Z(x)dx = \int_0^z \mathcal{P}_R(f(x))f'(x)dx = \int_0^{f(z)} \mathcal{P}_R(t)dt \,. \qquad (2$$

must hold for any $z \geq 0$ (here f' is the derivative of f). (2) shows how matching the percentiles of the density functions $\mathcal{P}_Z$ and $\mathcal{P}_R$ allows one to compute $f(z)$. This result is quite different from that of the conditional-mean approach of the previous section, because the fundamental a priori hypotheses are different in the two cases (indeed, in this second case, the joint distribution of (Z, R) is entirely supported on the 1-dimensional curve $R = f(Z)$ i he ZR-plane)

In practice, however, it will rarely be possible to classify the rain events so comprehensively as to end up with rain regimes each having an exact 1-1 Z–R relation. A small amount of uncertainty, due to intrinsic ambiguity in one's measurements, will inevitably remain. How does that affect the accuracy and applicabili y of the Z–R ela on obtained using formula (2), derived under idealized assumptions ?

To address this question, suppose that instead of an exact relation $f(Z) = R$, one postula es the existence of a relating function F which produces a "noisy" Z–R rela ion.

Mathematically, the assumption is that there exists an increasing function F such that

$$F(Z_d) = R_d + N \tag{3}$$

where N is a random variable representing additive noise, and where, to justify the additivity of this source of residual randomness, we use the dB-variables $Z_d = 10\log_{10}(Z)$ and $R_d = 10\log_{10}(R)$, instead of Z and R themselves. Given actual data, if one now computes a relating function $\hat{F}$ according to formula (2), i.e. a function which satisfies

$$\int_{-\infty}^{z} \mathcal{P}_{Z_d}(x)dx = \int_{-\infty}^{\hat{F}(z)} \mathcal{P}_{R_d}(t)dt \tag{4}$$

how close will this $\hat{F}$ come to the actual (optimal) Z-R function F?

Under the above hypothesis (3),

$$\int_{-\infty}^{z} \mathcal{P}_{Z_d}(x)dx = \mathrm{pr}\{Z_d < z\} \tag{5}$$

$$= \mathrm{pr}\{F(Z_d) < F(z)\} \tag{6}$$

$$= \mathrm{pr}\{R_d + N < F(z)\} \tag{7}$$

$$= \int_{-\infty}^{F(z)} \mathcal{P}_{R_d+N}(t)dt \tag{8}$$

Putting (4) and (8) together, this implies that $\hat{F}$ and F are related by

$$\int_{-\infty}^{\hat{F}(z)} \mathcal{P}_{R_d}(t)dt = \int_{-\infty}^{F(z)} \mathcal{P}_{R_d+N}(t)dt \tag{9}$$

To get a quantitative answer, let us make the simplifying assumption that R_d and N are independent, and that they are both Gaussian, with N having 0 mean. To be more exact, one could consider the presence of an additional reflectivity-noise term, i.e. replace (3) with $F(Z_d + N_Z) = R_d + N$, where N_Z and Z are independent. For simplicity, we shall assume that the radar-reflectivity measurements are accurate enough that N_Z would be small compared to the other terms and may therefore be ignored. Equation (9) then implies that

$$\hat{F}(z) = F(z) - \left(1 - \frac{1}{\sqrt{1+2\alpha}}\right)(F(z) - \mathcal{E}\{R_d\}), \tag{10}$$

where $\alpha = 0.5\sigma_N^2/\sigma_{R_d}^2$ is one-half the ratio of the variances of N and R_d, and where $\mathcal{E}\{R_d\}$ denotes the average dBR in this rain regime. Thus $\hat{F}$ under-estimates F at above-average rain rates, and over-estimates F at below-average rain rates, by amounts that are proportional to the ratio α of the noisy variation to the true variation. Converting back from logarithmic quantities, if we write $F = 10\log_{10}(f)$ and $\hat{F} = 10\log_{10}(\hat{f})$, and rewriting (10) in terms of the ideal f and the retrieved $\hat{f}$, one finds that the relative error is

$$\frac{\hat{f} - f}{f} = \left(\frac{\bar{R}}{f}\right)^{1-\frac{1}{\sqrt{1+2\alpha}}} - 1 \tag{11}$$

where $\bar{R}$ is the average rain rate in this rain regime. Thus, if the noisy variation is 20% as big as the true variation, i.e. if $\sigma_N = 0.2\sigma_{R_d}$ (so that $\alpha = 0.02$), in order for the relative error to exceed 5%, the ratio $\bar{R}/f$ must either fall below 0.08, or it must exceed 11. This means that even with a random variation that is 20% as big as the true variation in the rain rate, the relative error incurred in using the Z-R relation given by the "probability-matching" formula (2) will not exceed 5% at rain rates that lie in the interval $[0.1\bar{R}, 10\bar{R}]$ about the average rain rate.

This approach is the one adopted by Rosenfeld et al (1994) to implement the "probability matching method" (PMM). The classification criteria used in the PMM are

a) the effective efficiency, i.e. the relative difference between the cloud top and cloud bottom vapor saturation mixing ratios

b) the bright band fraction, i.e. the fraction of the radar echo area in which the maximal reflectivity occurs within ±1.5 km of the 0°C-isotherm

c) the horizontal reflectivity gradients

d) the freezing level itself.

The Z-R relation obtained as described above using these classification criteria has yielded quite accurate estimates of the near-surface rain rate for tropical rain systems near Darwin, Australia, as well as for winter convective rain systems in Israel. A detailed discussion of the results is beyond the scope of this paper. We refer the interested reader to (Rosenfeld et al, 1995a and 1995b) for a detailed discussion of the classification criteria and the resulting relations. Here, we briefly illustrate the method using the example of Fig. 1. Since the sample size is small, rather than use a comprehensive objective classification process, we start by (subjectively) classifying the data and retaining the data subset consisting of those measurements which correspond to stratiform rain. Figure 2a shows the conditional-mean estimate for the stratiform-rain data subset, obtained using (1) of the previous section, along with the two regression curves $Z = 100R^{1.47}$ and $Z =$ I 70/{ ''''7. Figure 2b shows the corresponding relative r.m.s. uncertainty. If we now assume that the classification in this case was tight enough to allow one to expect a nearly only one Z-R relation, the optimal estimate is the one given by the probability-matching formula (2). Figure 3 shows the resulting PMM relation, with the stratiform-rain power-law regression and the corresponding conditional-mean estimate overlaid. It can be noted that the regression seems to underestimate the rain-rate associated to higher reflectivities while it overestimates the rain-rate associated to lower reflectivites. The difference is not great because the sample size is very small indeed, but, as is shown below, this apparent trend is indeed confirmed by the theoretical analysis.

Figure 4a shows a contour plot of the percentage error that the probability-matching relation would be making, as a function of a) the ratio σ_N/σ_R of the noisy variation in the rain measurements to the true variation of the rain, and b) the ratio $R/\bar{R}$ of the rain rate to the mean rain rate, as given by (11). As observed above, as long as σ_N/σ_R is less than 20%, the error is less than 5% for $0.1 < R/\bar{R} < 10$. Figure 4b shows the ratio of the r.m.s. scatter that is due to residual noise to the true r.m.s. variation in the rain. Figure 4c shows the difference between 4a and 4b, i.e. the reduction of error when using the PMM.

Since the probability-matching relation is only an approximation to the optimal relation when σ_N is non-zero, one might well ask how well the power-law relation would do if it were used in this case instead. Since the power-law regressions are themselves only approximations of the conditional-mean Z–R relation, let us examine what the latter does in this case, *assuming that we are able to obtain simultaneous co-located (Z,R) measurements without introducing any additional error beyond the assumption of equation* (3), i.e. assuming that the scatter in the joint (Z,R) samples is due entirely to (3). This last condition is important because, on the one hand, it is practically impossible to realize, and because it is not a condition that one needs to worry about when using the probablity-matching relation: in that case, the individual marginal densities are all that is needed. Adopting the approach of section 2, (3) then implies that the joint density $\mathcal{P}_{(Z_d,R_d)}$ is given by

$$\mathcal{P}_{(Z_d,R_d)}(z,r) = \mathcal{P}_{R_d}(r)\cdot\mathcal{P}_N(F(z)-r)F'(z) \tag{12}$$

With the simplifying assumption that R_d and N are independent Gaussian with N having 0 mean, this would then imply that the conditional density function $\mathcal{P}_{R_d|Z_d=}$. is itself Gaussian with conditional mean

$$\mathcal{E}\{R_d|Z_d=z\} = \frac{F(z)+(\sigma_N^2/\sigma_R^2)\mathcal{E}\{R_d\}}{1+(\sigma_N^2/\sigma_R^2)} \tag{13}$$

and standard deviation $\sigma_N/\sqrt{1+(\sigma_N/\sigma_{R_d})^2}$. Thus the approach of section 2 would produce the Z–R relation given by the conditional mean

$$\hat{F}_{cm}(z) = \frac{F(z)+2\alpha\mathcal{E}\{R_d\}}{1+2\alpha} \tag{14}$$

where we have used α as in (10) to denote one half the ratio $\sigma_N^2/\sigma_{R_d}^2$ of the variances of N and R_d. Equation (14) shows the bias between the conditional-mean relation $\hat{F}_{cm}$ and the actual relation F, while the r.m.s. uncertainty in the relation $\hat{F}_{cm}$ is actually $\sigma_N/\sqrt{1+(\sigma_N/\sigma_{R_d})^2}$. Upon converting back from logarithmic quantities, the relative bias between this relation and the underlying actual relation f is

$$\frac{\hat{f}_{cm} - f}{f} \simeq \left(\frac{R}{f}\right)^{-\frac{2\alpha}{1+2\alpha}} - 1, \tag{15}$$

which is always bigger in absolute value than the relative error (1 1) in the probability-matching approach (because $2\alpha/(1+2\alpha)$ is always bigger than $1 - 1/\sqrt{1+2\alpha}$). 01" ('0111's('. the error in the conditional-mean Z-R relation will increase further i{' one replaces this relation by its corresponding regression-based power-law approximation.

It may seem surprising that the conditional-mean approach does not choose the relation f itself as the way to estimate R, but rather an exaggerated "bias" as shown in (14). The reason for this apparent discrepancy is the following: when the noise variance is zero, (14) confirms that the conditional-mean method does use f to obtain its estimate of the rain, as expected; however, as the noise increases, the conditional-mean approach biases its estimate towards the average $\mathcal{E}\{R\}$, until, in the limit, when the noise variance tends to infinity, it (properly) rejects the Z-R relation altogether and prefers to estimate R by its a-priori-known mean $\mathcal{E}\{R\}$ without any regard to the useless value of Z. This bias is responsible for the fact that the resulting relative difference between $\hat{f}_{cm}$ and the underlying f is greater than the difference between the probability-matched $\hat{f}$ and f. In the power-law regressions, it would translate into an exaggerated underestimate of the rain-rate associated to higher reflectivities, and an overestimate of the rain-rate associated to lower reflectivities.

4 Conclusions

The two empirical methods of deriving Z-R relations, power-law regressions and the PMM-based approach, are not directly comparable because they start with fundamentally different underlying mathematical assumptions about the nature of the randomness in the data. The power laws are approximations to the optimal relations when the original data is largely uncategorized (in that case, the seldom-used conditional-mean method actually gives the optimal relation). The PMM is the optimal relation when the original data is classified a priori in such a w 'ii)' that one may reasonably ('X])('ct a one to one ("0l'l'('s!)01l(l('1l(" ('" between Z and R. The PMM remains a better approximation than the conditional-mean method (and, a fortiori, better than the regression-based power laws) if a relatively small amount of residual randomness is still present after classification, i.e. if, up to some measurement white noise that is small compared to the rain intensity, one can still expect that one's classification produces categories each admitting a one-to-one correspondence between Z and R.

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Figure captions

Figu e a: Convective and stratiform rain regimes of tropical squall lines in Darwin, Australia Reflectivity factor versus rain rate observed on 26 January 1989, between 1800 and 2100 local time (after Short et al, 1993).

Figure b: Conditional mean relation for the data of figure 1a. The dashed lines correspond to the two power-law regressions $Z = 170R^{1.47}$ and $Z = 400R^{1.47}$ corresponding to the two rain regimes governing different segments of the data.

Figure 1c: Relative r.m.s. uncertainty in the conditional-mean relation of figure 1b, in %, as estimated by the ratio of the condi ional standard deviation to the conditional mean.

Figure 2a: Conditional-mean relation for the subset of the data of figure 1a corresponding to stratiform rain.

Figure 2b: Relative r.m.s. uncertainty in the conditional-mean relation of figure 2a, in %

igu 3 Probability-ma ched relation for he subset of the data of figure 1a corresponding to s a iform ain The lashed lines correspon to the power law regression $Z = 400R^{1.47}$ an the conditi nal-mean rela ion.

igure 4a: Contour plot of the relative error made by PMM, in %, as a function of the ratio σ_N/σ_R of noisy variation in the measruements to true variation in the rain (in %), and of the ratio $10\log_{10}(R/\bar{R})$ of the rain rate to the mean rain rate (in dB) - see (11).

Figure 4b: Contour plot of the ra io σ_N/σ_R of noisy variation in the measruements to true variation in the rain, in %

Figure 4c: Contour plot of the difference between the relative r.m.s. variation of the noise and the relative error made by PMM, in %

Figure 1 a

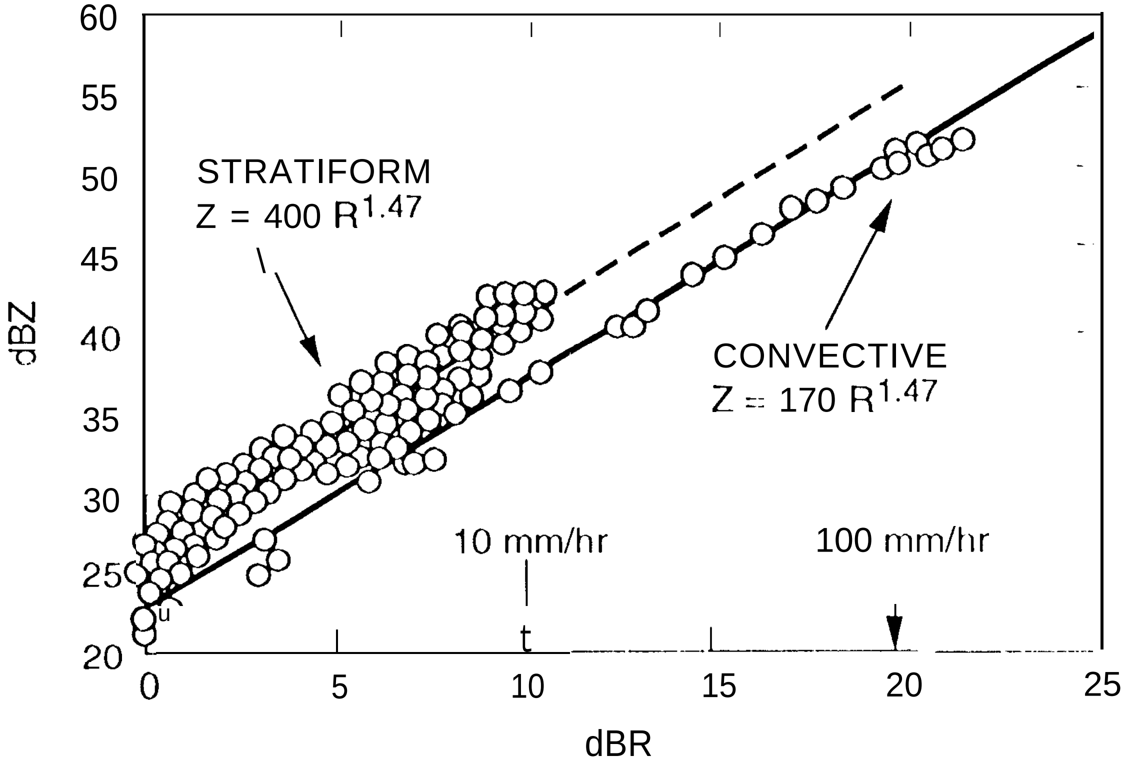

STRATIFORM
Z = 400 R1.47
CONVECTIVE
Z = 170 R1.47
10 mm/hr
100 mm/hr
dBZ
dBR
60
55
50
45
40
35
30
25
20
0
5
10
15
20
25

Figure 1b

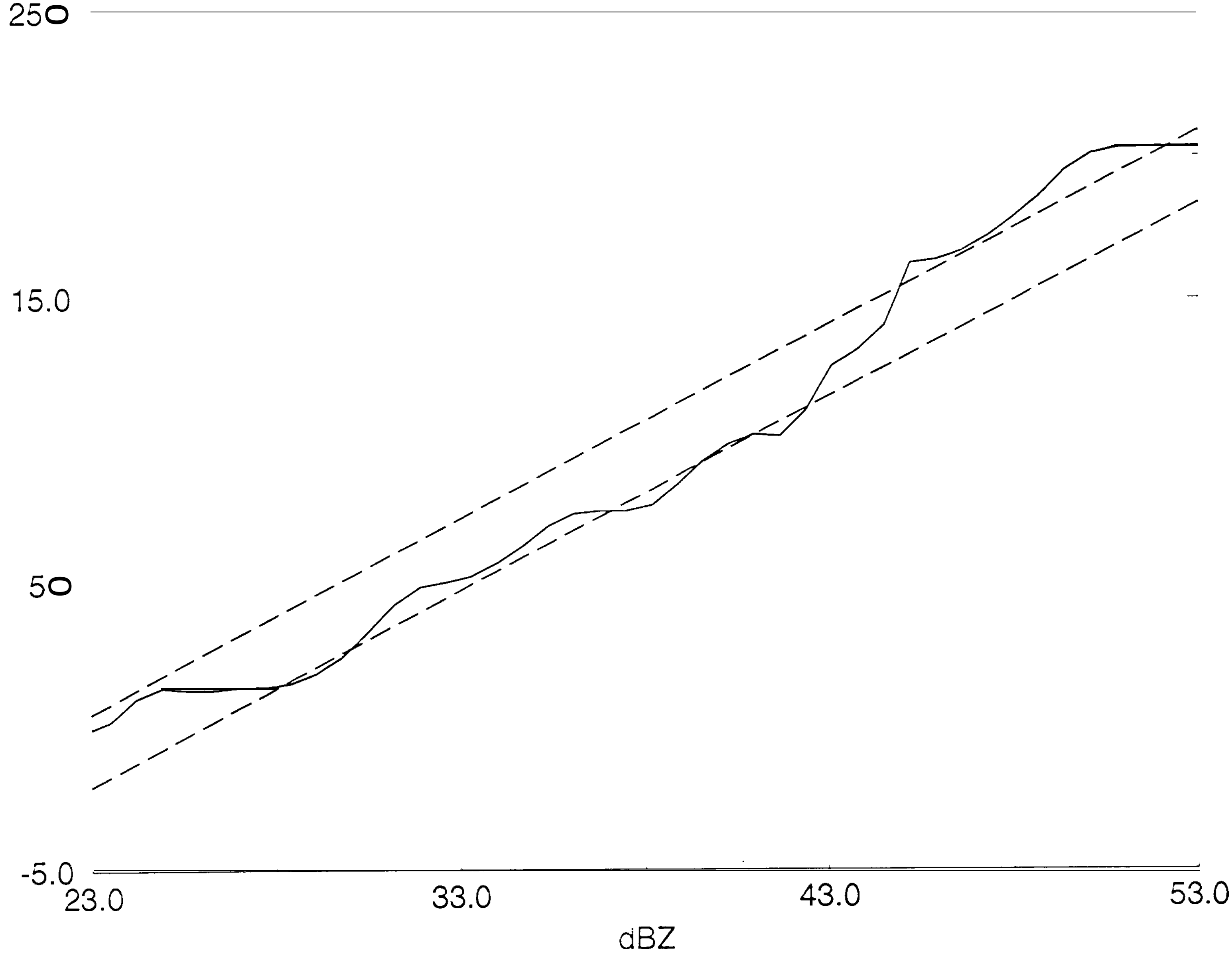


Figure 1c

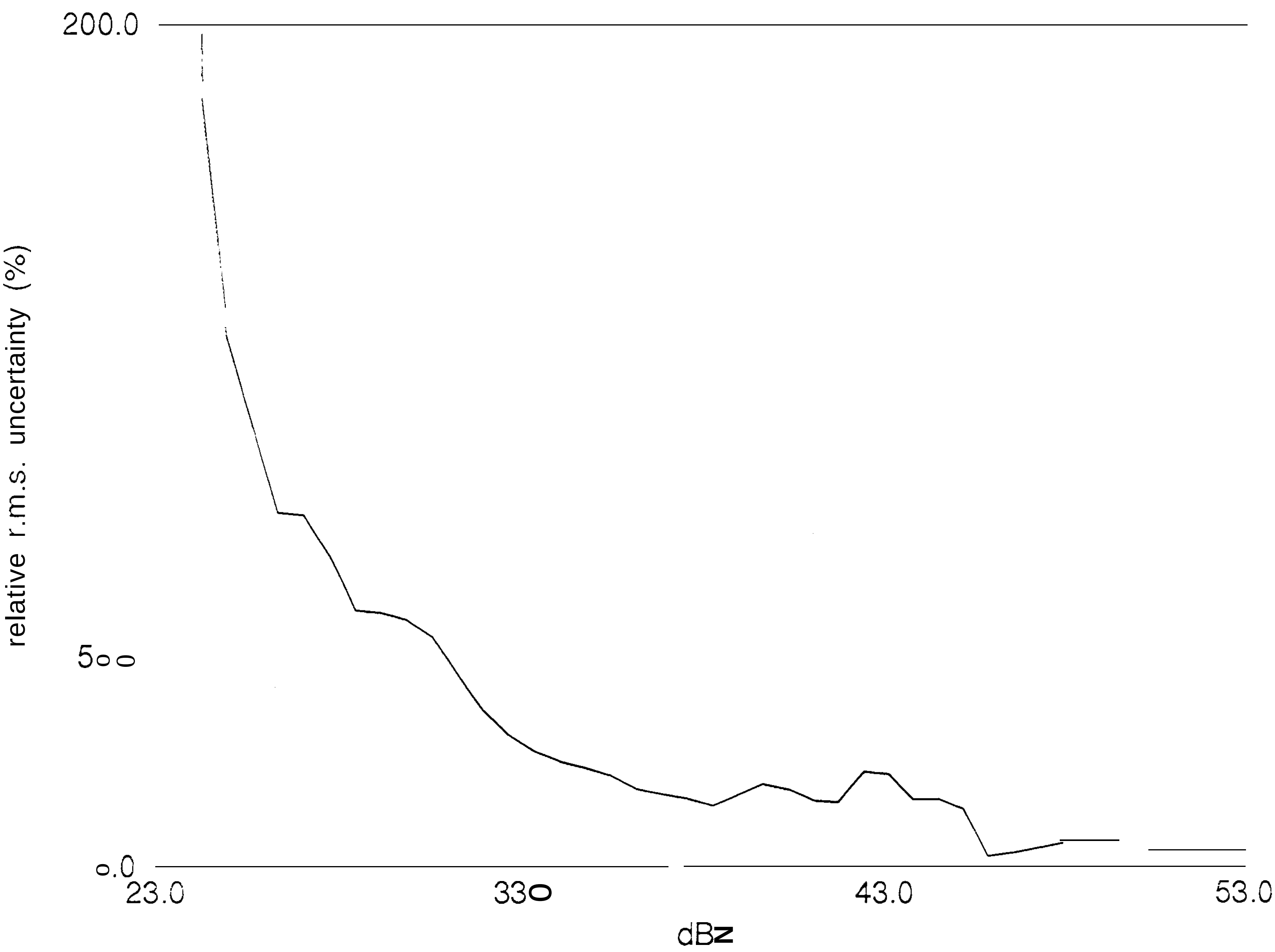


Figure 2a

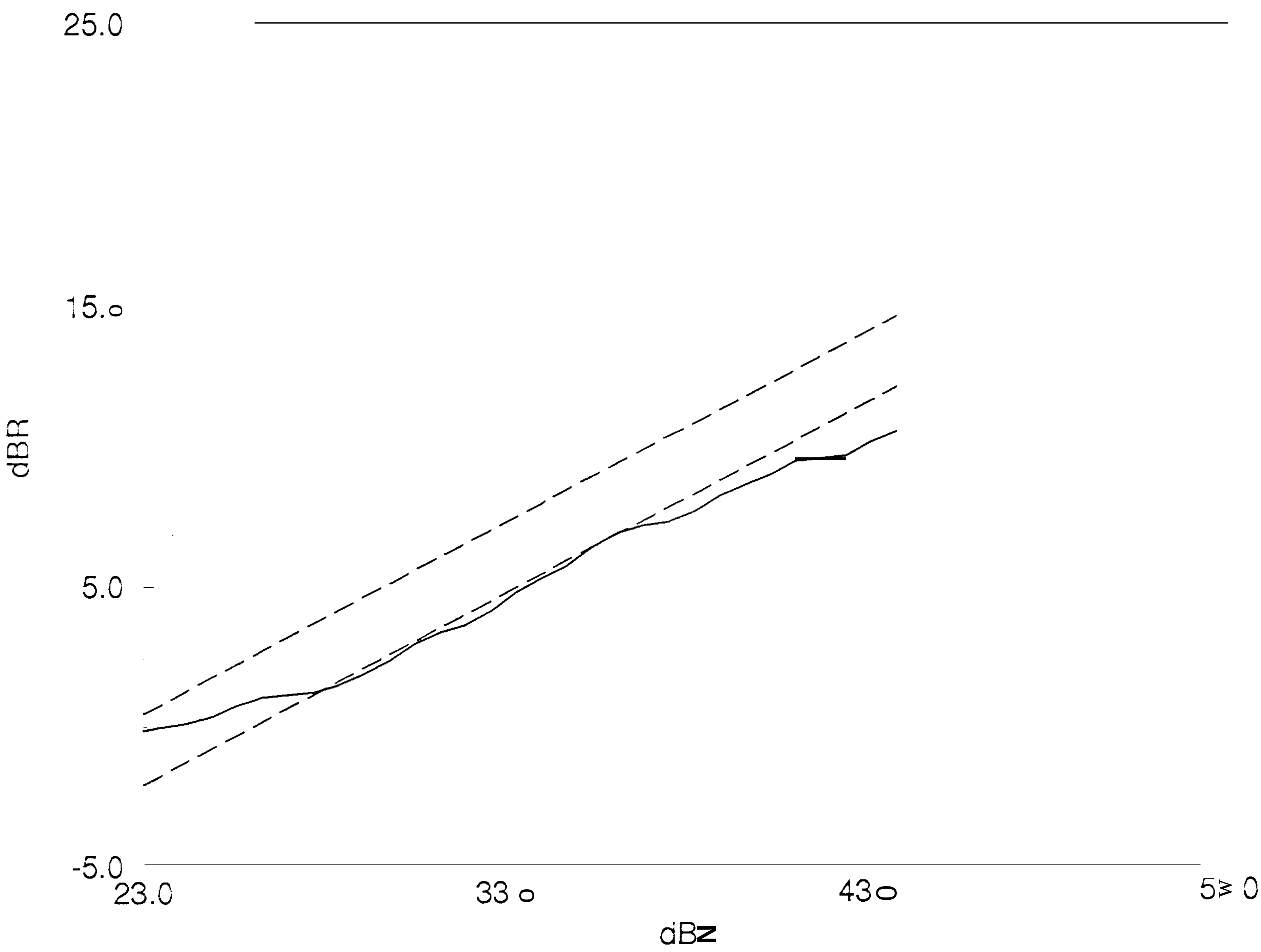


Figure 2b

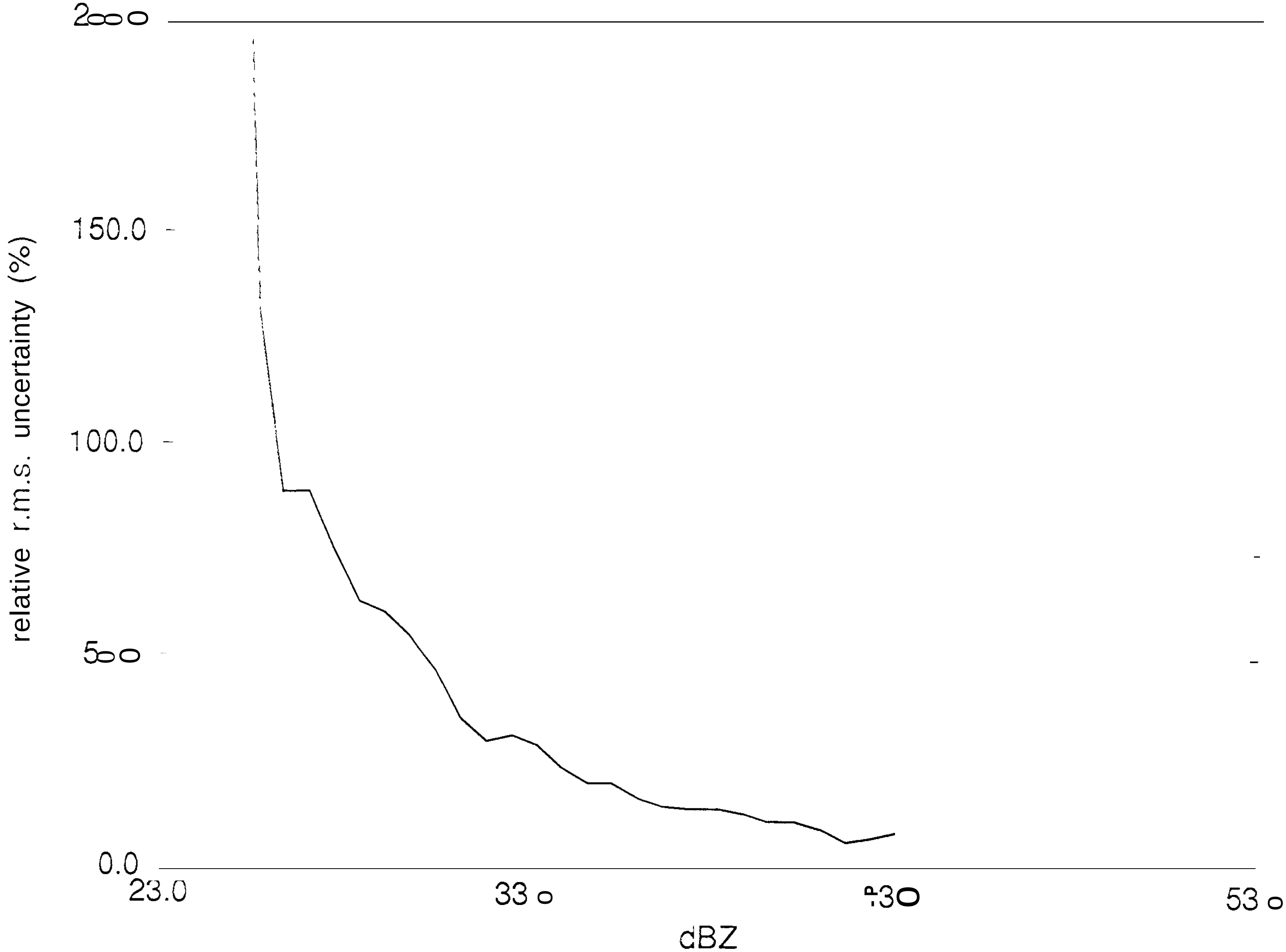
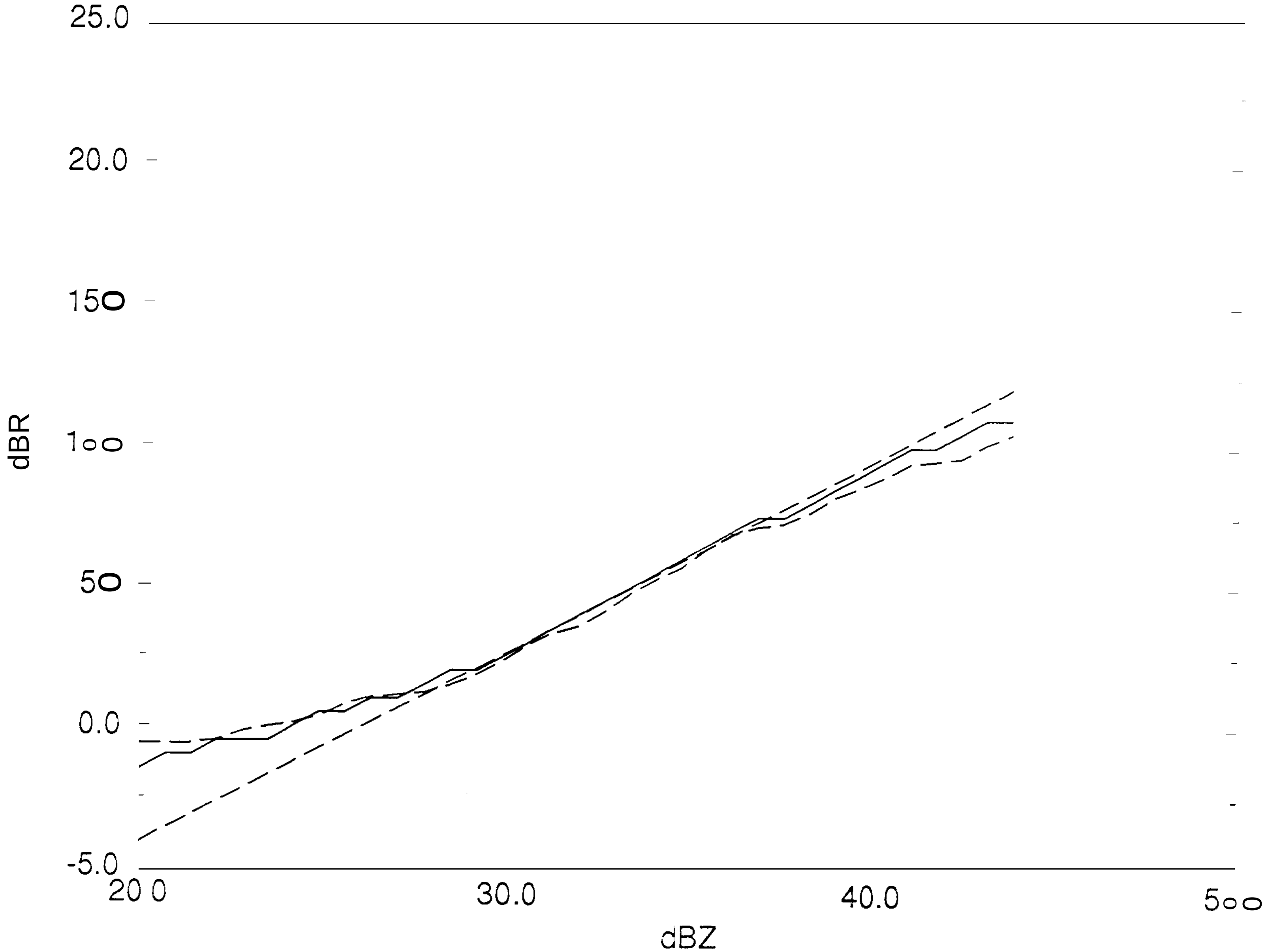

relative r.m.s. uncertainty (%)
200.0
150.0
100.0
50.0
0.0
23.0
33.0
43.0
53.0
dBZ

Figure 3



ig ure 4 a

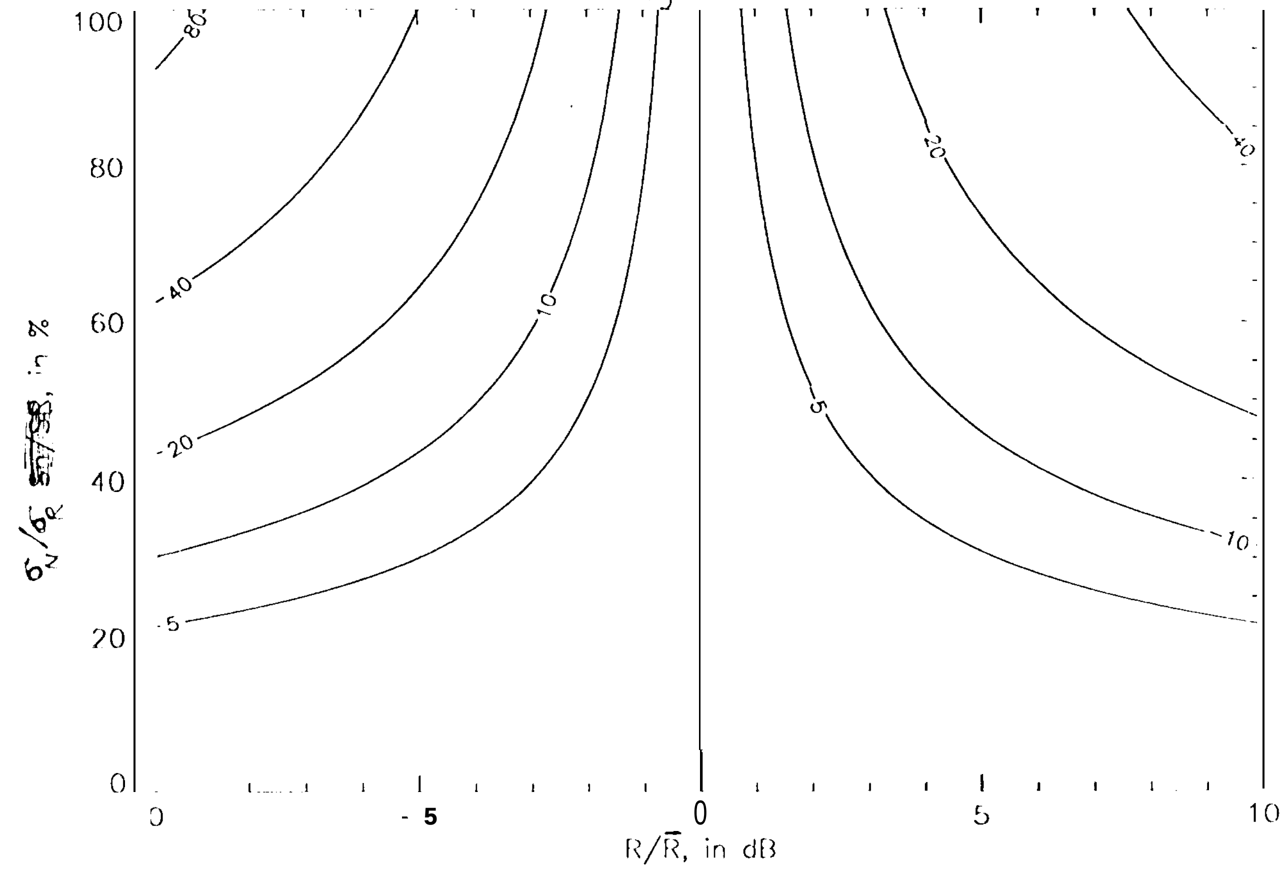


Figure 4b

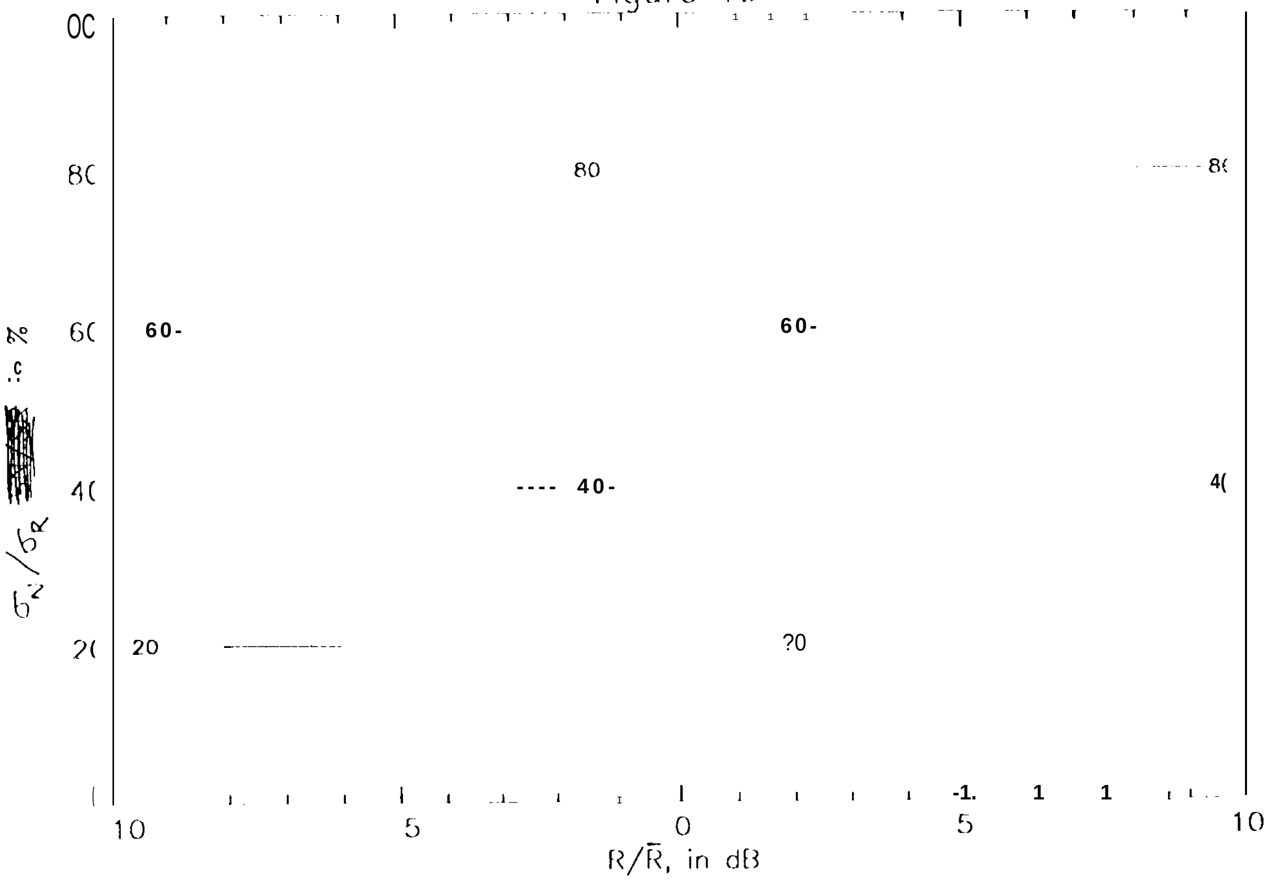


Figure 4c